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Use of the kriging method in determining the properties of gases in large channels

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ABSTRACT

Properties of fluids can vary significantly over a cross-section of a channel. The variations can be graphically presented by means of profiles which are typically based on a limited number of measured points. The article presents suitability of the kriging interpolation method for data analysis at measuring properties of fluids in large channels in various thermal and process plants. Using a practical example rather than complex statistical analyses the advantages of the kriging over other interpolation methods are presented. Several examples also give some guidelines on choosing number and distribution of measuring points to ensure accurate profiles.

1. Introduction

Measurement of the parameters of gases or liquids in power plants is important for monitoring the operation of the entire plant, both from the viewpoint of energy efficiency and reliability of operation, and regarding its influence on the environment. In channels and pipes of larger dimensions, the values of a certain parameter may be quite non-uniform over the channel's cross-section. Regardless of whether only the average value of the parameter is important or the parameter's variation over the cross-section is investigated, in such cases the value of the parameter needs to be measured in a bigger number of points [1,2]. Because of the way measurements are performed, the number of measurement points is limited, therefore even grid measurements cannot provide data on the value of the measured parameter at any particular point of the channel's cross-section. However, on the basis of the measured values, it is certainly possible to estimate the value of the chosen parameter in places where measurement was not done. Several interpolation methods have been developed for this purpose. While linear interpolation is the least complex interpolation method it does require appropriate tessellation of the area where calculations are to be performed. For planar cases Delaunay triangulation is commonly used [3,4]. Polynomial regression can also be used to find values of the chosen parameter where measurement was not or could not be performed [5]. In the field of geostatistics, the kriging method is often used, in which the unknown value of a parameter is calculated as the weighted average of known, measured values [6–9]. In this manner it is possible to determine the value of a studied parameter at any chosen number of points within a certain channel cross-section. By calculating parameter values in a sufficiently large number of points over the entire cross-section, the so-called profile of parameter variation across the entire cross-section is also obtained. All the mentioned methods have advantages and disadvantages regarding the particular case where they are applied. Linear interpolation and kriging retain values of the observed parameter in the points where the values are measured and they can be assumed to be correct. Polynomial regression on the other hand causes some 'smoothing' of the profile depending on the polynomial function used and the actual profile of the parameter. Linear interpolation and kriging perform best within the region of the available data while polynomial regression can easily be extrapolated beyond the region of the measurement points.

This paper briefly presents the procedure for planar interpolation using the kriging method, along with its use on an example of determination of flue gas properties over channel cross-section behind a rotational air heater. The conditions within the channel are estimated by numerical simulation. A combined CFD and regenerative heat transfer model was used to simulate three-dimensional velocity, pressure, temperature and gas composition fields within a rotary air heater and the adjoining flue gas channels [10,11]. The actual profiles are therefore assumed to be thoroughly

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Nomenclature

d	distance
i, j	integer counter
K	constant
l	lag distance
N	number of data points
n	number of point pairs within lag
P	computational point
T	data points
w	weight
$\mathbf{w}$	vector of weights
x,y,z	Cartesian coordinates
γ	general function
ϕ	general parameter

known, and the results obtained using the kriging method can be compared with them.

2. Observed planar profile

For illustrative presentation of characteristics of determination of planar parameter profiles using kriging and polynomial regression method, a case of temperature profile of flue gases in a channel with sides of 8 m and 3.6 m that is located behind a rotational heat exchanger was used. The temperature profile was calculated by means of numerical simulation of the operation of rotational heat exchanger. Fluid flow simulations are based on solving a system of transport equations which is done with commercial CFD software. Additional model was used to simulate regenerative heat transfer within the matrix of the heat exchanger [10,11]. The simulation yields the values of flue gas temperature throughout entire three-dimensional computational domain including the studied plane. The virtual measuring plane thus consists of almost 4800 points where the observed parameter is known. Fig. 1 depicts the reference temperature profile on the measuring plane. This data will be used for comparison with the values calculated with spatial interpolation. Measurements within the channel are substituted with sampling a certain number of points on the virtual measuring plane; these represent known points T_i. The temperatures at the selected points thus represent known values of parameter ϕ_i. All profiles which will be presented in subsequent sections are shown in a comparable manner and using the same temperature scale as in Fig. 1.

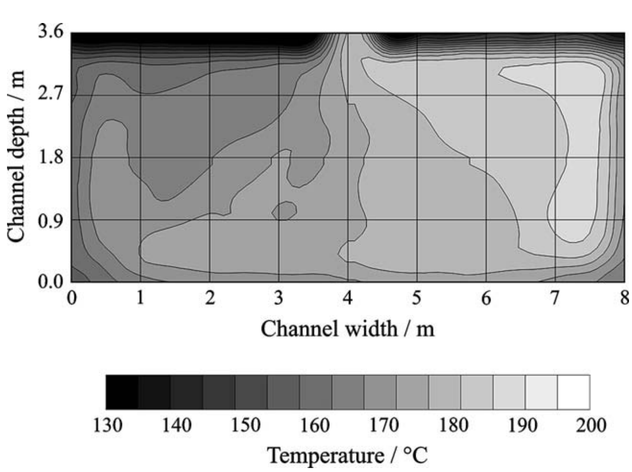

Fig. 1. Actual profile of the studied parameter – flue gas temperatures behind a rotational heat exchanger.

3. Spatial interpolation using the kriging method

In engineering practice and research, parameter measurements are often required. The values of such parameters may continuously vary within a certain space or along a plane. Measurements are usually performed in discrete points and the number of measurement points is limited by the available equipment and time. Furthermore, the locations of measurement points usually cannot be optional, because they are determined by the design of duct and accessibility of the area in which measurements are done. Even in places where individual parameter values are not known for any reason, these can be determined on the basis of known values at other points. For this purpose, an interpolation method is necessary that will take into account as many known points as possible and will therefore include the characteristics of variation of the studied parameter across the entire measurement range.

In the fifties of the previous century, the South African mining engineer Danie G. Krige studied a similar problem. On the basis of a limited number of soil samples, he tried to determine the content of a certain ore at sites from which he had no available samples. French mathematician Georges Matheron further developed the interpolation method proposed by Krige and named it kriging [6,8]. Nowadays, this method is used mostly in geostatistics, but it is also suitable for the analysis of measurements in various fields of engineering, for example in power and process plants [2,12,13].

In the case of the kriging method, estimation of the value of any chosen parameter $\phi = \phi(x,y,z)$ in a computational point $P = (x,y,z)$, where its value is unknown, is based on the known values ϕ_i of the same parameter in N data points $T_i = (x_i,y_i,z_i)$ in the vicinity of point P. The estimate of the sought value is the weighted average of all known values [6,8].

$$\phi = \sum_{i=1}^{N} w_i \phi_i \tag{1}$$

Factors w_i are weights which represent the influence of individual data point T_i on the value of parameter ϕ at point P. All weights together compose the vector of weights $\mathbf{w}$ for point P. The sum of vector components i.e. all weights w_i should equal 1 to guarantee uniform unbiasedness of the estimated values [8], i.e. the average value of the actual parameter values is the same as the average value of the estimated parameter values [6].

$$\sum_{i=1}^{N} w_i = 1 \tag{2}$$

3.1. *Weights for calculation of parameter values in computational points*

The values of weights for individual points are determined on the basis of the assumption that an unknown value of parameter ϕ at point P is more likely to be similar to the values at points T_i close by than in more remote points. The weights of points which lie closer to the computational point P will therefore be higher, while those for more remote points will be lower, possibly even negligibly small [6]. The influence of distance can generally be described using appropriate function $\gamma(d_{i,j})$ in which $d_{i,j}$ represents the distance between points i and j. The optimal weights are calculated using the following system of linear equations:

$$
\begin{aligned}
&w_1 + w_2 + \cdots w_N = 1 \\
&w_0 + w_1\gamma(d_{1,1}) + w_2\gamma(d_{1,2}) + \cdots + w_N\gamma(d_{1,N}) = \gamma(d_{1,P}) \\
&w_0 + w_1\gamma(d_{2,1}) + w_2\gamma(d_{2,2}) + \cdots + w_N\gamma(d_{2,N}) = \gamma(d_{2,P}) \\
&\vdots \\
&w_0 + w_1\gamma(d_{N,1}) + w_2\gamma(d_{N,2}) + \cdots + w_N\gamma(d_{N,N}) = \gamma(d_{N,P})
\end{aligned}
\tag{3}
$$

Besides the weights $w_1,\ldots,w_N$ additional variable w_0 was added to the system to ensure that the system will only have one solution [8].

3.2. Determination of function γ(d)

The selection of the function to describe the distance between individual points $\gamma(d)$ is based on the squared difference of parameter value ϕ at two different points. Some guidelines for selecting appropriate function and criteria for their evaluation are described in [14]. Kriging assumes that in general the difference of parameter value in two selected points increases with increasing distance between the two points. If a chart is created with distance between the points (lag, l_i) on abscissa and the squared difference (γ_i) on ordinate an increasing function as shown in Fig. 2 can be observed [6].

The discrete points are approximated using the best fitting function $\gamma(d)$, which is called the model variogram; one of the following models usually proves appropriate [8]:

- Linear: $\gamma(d) = K_1 \cdot d + K_2$
- Gaussian: $\gamma(d) = K_1 \cdot \left(1 - e^{-d^{K_3}}\right) + K_2$
- Spherical: $\gamma(d) = \begin{Bmatrix} K_1\left(1{,}5d/d_N - 0{,}5\left(d/d_N\right)^3\right) + K_2 & ;d \le d_N \\ K_1 + K_2 & ;d > d_N \end{Bmatrix}$

d_N is the limit distance between two points, onward from which variances γ_k randomly oscillate around a constant value called the 'nugget', Fig. 2

- Power: $\gamma(d) = K_1 \cdot d^{K_3} + K_2$

The parameters K_1, K_2 and K_3 in the selected model are adjusted such that the model variogram best fits the calculated points γ_k.

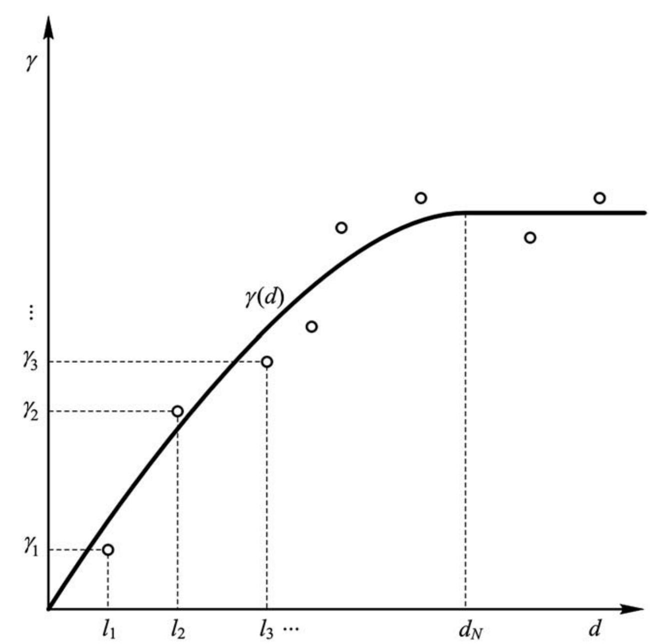

Fig. 2. Calculated variances and spherical model variogram.

Depending on the actual case different fitting approaches can be used, e.g. least squares and it's derivatives are presented in [5].

Once the model variogram has been selected and the parameters K_1, K_2 and K_3 have been determined, the linear system of equations (3) is solved using the Gaussian elimination method. The obtained result is the vector of weights $\mathbf{w} = (w_1, w_2, \ldots, w_N)$ which are used in (1) for calculation of the value of parameter ϕ at point P. Fig. 3 shows a schematic presentation of this procedure.

4. Planar parameter profile

Instead of a single point P at which one would like to determine the value of parameter ϕ, it is sometimes necessary to know the variation of this parameter over the entire computational range, for example the measurement plane. The kriging method described above is used for calculating the value of the parameter in a bigger number of points P_j, which, for the presentation purposes, usually lie at the nodes of an orthogonal grid. The variogram and the model variogram $\gamma(d)$ depend only on the parameter's value and locations of the known points and therefore remain unchanged in computations for all grid points. The vector of weight $\mathbf{w}_j$ for calculating the parameter's value at the computational point P_j, however, depends on the point's location and its distance from the known points T_i, therefore the entire vector needs to be calculated separately for each point. By using a computer and appropriate numerical methods for solving systems of linear equations, it is possible to quite quickly calculate the values of the parameter even for a relatively large number of points.

For the presentation of certain characteristics of determination of planar parameter profiles using the kriging method, we used the case of the temperature profile of flue gases in a channel with sides of 8 m and 3.6 m that is located behind a rotational heat exchanger. The actual temperature profile consisting of curves of constant values (Fig. 1) was calculated by means of numerical simulation of the operation of rotational heat exchanger [10]. This yields the actual values of temperature at almost 4800 points on the studied plane and can be used for comparison with any values that are calculated later.

Channel measurements are substituted with a certain number of points taken from the profile obtained by numerical simulation; these represent known points T_i. The temperatures at the selected

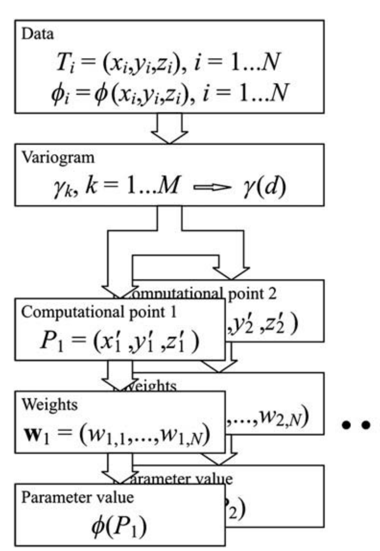

Fig. 3. Schematic presentation of the computational procedure according to the kriging method.

points thus represent known values of parameter ϕ_i. All profiles which will be presented in the figures below are shown in a comparable manner and using the same temperature scale as in Fig. 1.

4.1. Comparison of kriging and polynomial regression

Instead of the kriging method, other methods can also be used to illustrate the profile of parameter ϕ on a certain plane. Using polynomial regression, function $\phi(x,y)$ can be determined, with which one can determine the value of the parameter at any point of the computational plane. Two samples are selected from the profile shown in Fig. 1, one comprising 32 points and another comprising 60 points, and they are used to estimate the temperature profile. Fig. 4 presents the profiles obtained using the kriging method (examples a) and b)) and also using polynomial regression (examples c) and d)). Variogram from the available data showed that best model variogram is spherical model with parameters $K_1 = 240$, $K_2 = 0$ and $d_N = 4$, see Section 3.2, which was used for kriging. In the case of polynomial regression, the variation of temperature with the point's position in the channel is expressed with the following function:

$$T(x,y) = K_1 + K_2x + K_3y + K_4x^2 + K_5xy + K_6y^2 + K_7x^3 + K_8x^2y + K_9xy^2 + K_{10}y^3 \quad (4)$$

All profiles in Fig. 4 show white areas, in which the deviation of the calculated profile from the actual one (Fig. 1) is lower than 0.5 °C. Table 1 contains data on the percentage of the total cross-section surface area that is covered by the white area in all four cases as well as average error for particular case. The average error is average value of absolute differences between actual and estimated value of the observed parameter in all available 4800 points.

Both Fig. 4 and data in Table 1 show obvious difference between the compared methods. The kriging method makes it possible to capture the non-uniformity of the temperature profile much better. Using the kriging method, the percentage of the computational surface area in which deviations of the studied parameter are relatively small is significantly higher than when polynomial regression is used. In addition, the greater the number of points, the higher is the above-mentioned percentage of the surface area in the case of the kriging method, while during polynomial regression even a reduction can be noticed, Table 1. When the kriging method is used, a higher-quality profile is achieved in the presented case that comprises 32 points than with polynomial regression that comprises 32 or even 60 points. However, a higher number of known (measured) points also results in a proportionally longer time or amount of equipment necessary to perform the measurements.

Table 1
Comparison of profile quality for profiles calculated using either kriging or polynomial regression.

		Kriging		Polynomial regression	
		a)	b)	c)	d)
Percentage of surface area where error is lower than 0.5 °C	%	26	31	12	10
Average error	°C	5.2	3.9	5.6	4.7

A disadvantage of the kriging method can also be seen in Fig. 4. Outside the region of the sample points estimates are a lot less accurate than inside the region. This characteristic of kriging should be taken into consideration while planning measurements to appropriately position the measuring points.

Another characteristic of the kriging method which can also be seen in Fig. 4 is that the values of the parameter at the known points remain unchanged, which generally does not happen with regression. Therefore, if one of computational points P matches a known point T_k, the kriging method yields values $\phi(P) = \phi(T_k)$ [8]. All of the known points along the profiles a) and b) therefore lie within the white area, while in the case of polynomial regression only a few known points along profiles c) and d) lie within the shaded area.

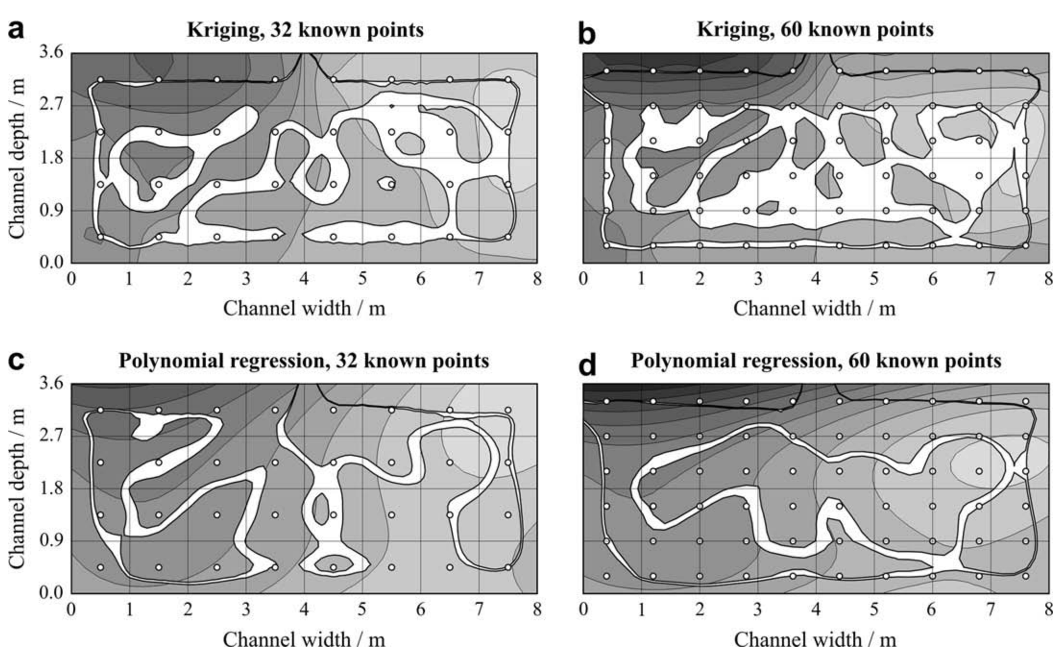

Fig. 4. Comparison of kriging and polynomial regression for determination of planar parameter profile.

4.2. Computational parameters for calculating planar profiles using the kriging method

The accuracy of the resulting profile of the studied parameter ϕ is also affected by the way the described method is used for calculating the planar profile. The temperature profile of flue gases as shown in Fig. 1 is again taken as the basis. Among all of the known points, some are selected for the sample, and from this sample a certain number of computational points are calculated and used to graphically present the studied profile. The quality of the calculated profile is significantly affected by:

-The number of known points (sample size), and
-The distribution of known points.

According to [15], in large ducts properties of fluids can only be obtained by measurement traverse with appropriate number of measuring points. Each of the measuring points should cover a rectangular area of less than 0.5 m^2, and the ratio of the area sides should be less than 2. The recommended number of measuring points thus depends on the size of the measuring plane. The locations of measuring points should be distributed over a homogeneous orthogonal grid with control areas as much as possible square shaped.

4.2.1. Sample size of known points

In order to be able to make a profile of a certain parameter over a selected measurement plane, it is necessary to first measure the values of this parameter at a certain number of known locations on the studied plane. The number of measurement points is directly related either to the amount of necessary measuring equipment (if measurements in all points need to be performed simultaneously) or with the duration of measurement (if measurement is performed by probing) separately in each point. Furthermore, the number of measurement points and their positions are also usually limited by the construction of the channel in which the measurements are performed. Fig. 5 shows the influence of the number of known (measured) points on the shape of the calculated profile. All profiles from Fig. 5 can be compared with the actual profile shown in Fig. 1, which is quite non-uniform and shows large gradients of the studied parameter. Therefore, it is obvious that this can be described sufficiently well only if a considerable number of measurement points are used. In the given case, even 60 measurement points are not enough to capture all of the details of parameter variation as shown in the actual profile. With merely eight points, it is possible to estimate only the chief trend of parameter variation, but they certainly do not yield a satisfactory profile over the entire measurement plane. Such a small number of measurement points do not suffice for determination of the parameter's profile on the studied plane, especially if the conditions are as non-homogenous as in the presented case.

4.2.2. Distribution of measurement points

In cases of asymmetric flow special attention should be put to the distribution of measuring points [16]. If there is a small number of known points, deviations from the actual profile along the channel wall are most obvious, primarily along the upper profile edge, where temperature gradients are the largest. This area already lies outside of the scope of known points, therefore in this place the parameter's value can be estimated only by extrapolation. Measurement points can be redistributed closer to the channel edge, and in the critical part (on the upper edge) a few additional measurement points can be added as well. Fig. 6a shows that mere redistribution of measurement points does not assure a more accurate profile over the entire computational range. More accurate information about the actual situation and therefore a more accurate profile are obtained along the channel walls, but a different distribution of known points notably changes the temperature profile in the central part of the channel.

The addition of eight known points, i.e. 40 in total (Fig. 6b), better describes the situation along the upper channel edge, while at the same time it does not cause any significant change of the profile in the central or lower part of the channel. However, an unusual shape of the profile along the upper profile edge can be noticed.

In this part of the channel, the surface areas belonging to individual known points have very high aspect ratio (proportion of the longer to the shorter side of the rectangle), while elsewhere they

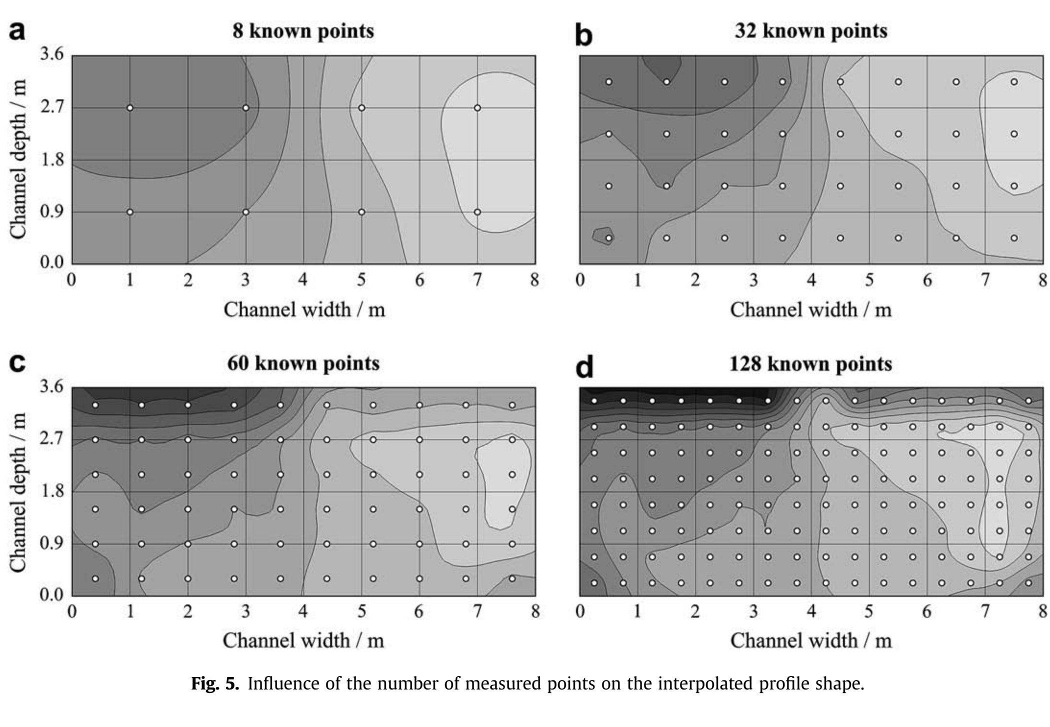

Fig. 5. Influence of the number of measured points on the interpolated profile shape.

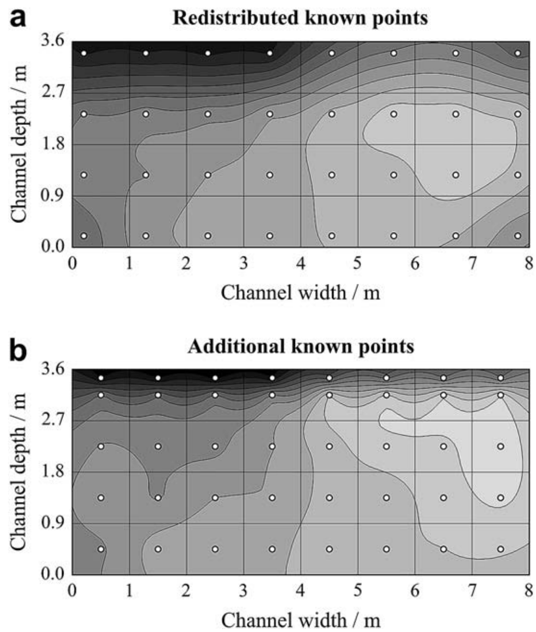

Fig. 6. Improvement of calculated profiles by changing the distribution of known points or by adding known points.

are almost quadratic. Therefore, the shape of the control surface covered by an individual measurement point also affects the shape of the calculated profile. This influence is shown in Fig. 7, where profile a) is recalculated using 32 known points, which are arranged in a different pattern over the cross-section than is seen in Fig. 5b. The aspect ratio in the case shown in Fig. 5b is 1.1, and in Fig. 7a it is 4.4. The resulting profiles are thus quite different.

Profile b) in Fig. 7 is calculated from only 16 known points. Their control areas of individual points have aspect ratio of 2.2, therefore the calculated profile is more similar to the middle profile in Fig. 5b than profile in Fig. 7a. During measurements, it is therefore important to make sure that the control surfaces are as quadratic as possible, to the extent that is permitted by the plant design.

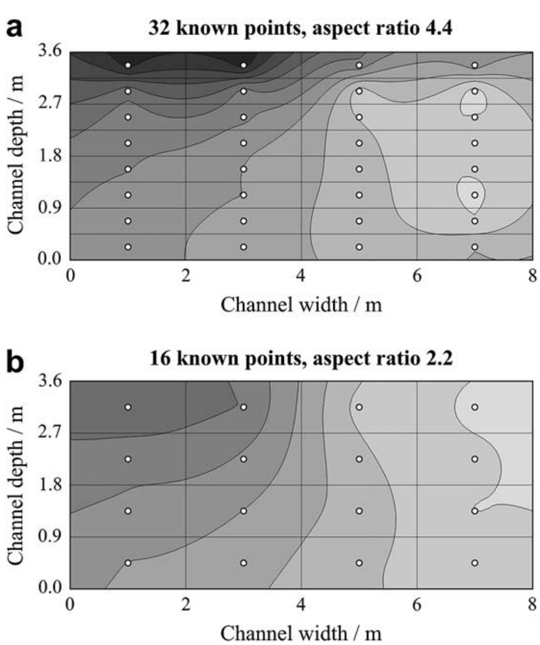

Fig. 7. Influence of aspect ratio on the calculated profile shape.

5. Conclusion

In engineering practice, one often encounters flows of gases or liquids with properties that vary over a certain cross-section. This non-homogeneity can best be demonstrated using planar profiles in which parameter values are graphically presented at all points of the studied plane. In practice, it is not possible to measure these values at every point of the cross-section, therefore it is necessary to estimate the parameter values across the entire cross-section on the basis of a limited number of known points. This paper presents the kriging interpolation method, along with an algorithm for determining the value of the studied parameter at any point of the cross-section. The applicability of the kriging method in power engineering is presented on a case of flue gas temperatures behind a rotational heat exchanger.

A comparison of the kriging method with polynomial regression is presented. The studied case shows that the temperature profile calculated using the kriging method matches the actual profile much better. It also turns out that the kriging method ensures better results even with a smaller number of known points than polynomial regression with a greater number of known points. An important advantage of the kriging method is also that the parameter values calculated for known points equals their actual values.

Subsequently, the influence of the number and distribution of known points on the quality of the resulting profile was analyzed for the kriging method. The number of known points has a significant influence on the shape and accuracy of the calculated profile. Although a greater sample of known points does increase the number of computational operations necessary for calculating the profile, it can describe the actual situation in the computational plane much better.

The number of known points, i.e. the number of measurement sites on the measurement plane, is usually limited by the plant design, available equipment and time. Therefore, in addition to the number of known points, their distribution is also very important. To be able to accurately present all of the characteristics of the actual profile, it is necessary to place the measurement points into those parts of the cross-section in which greater gradients of the studied parameter are found. The profile shape therefore needs to be assessed in advance, e.g. on the basis of experience, and computer simulation of circumstances within the computational range can also be useful for this purpose.

Concerning the distribution of the known points for the kriging method, it is also recommended to keep the distances between individual measurement points as uniform as possible. If the measurement points are arranged on an orthogonal grid, the surface area belonging to individual points should be as quadratic as possible.

In engineering practice, including measurements in the field of power engineering, the kriging method is a useful tool for the determination of planar profiles of various parameters. Its main advantages over other methods are its suitability for use on practically any set of known values and the fact that with this method the values of the parameter at known points remain unchanged. However, it depends on the particular problem how measurements to determine known points are to be performed and how the parameters for profile calculation should be selected so that the profile would present the actual situation on the studied cross-section as accurately as possible.

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